
BENCHMARKS IN LEIPZIG

A COLLECTION OF QUESTIONS IN RESEARCH-LEVEL MATHEMATICS

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ABSTRACT

Between April 1 and May 15, 2026, a group of 49 mathematicians compiled a dataset of research-level mathematics questions with known answers. Most of the work was done during the 3-day workshop **Benchmarks in Leipzig** with 35 participants at the Max Planck Institute for Mathematics in the Sciences in Leipzig, Germany. We present the resulting collection of 100 questions. We evaluated these questions in three stages: a single attempt by five state-of-the-art LLMs, followed by a 20-runs-per-model evaluation with three of these models, and finally a 3-run attempt with two heavy-thinking models. After Stage 1, 41 questions remained completely unsolved; after Stage 2, this count dropped to 16; and we concluded Stage 3 with only 2 unsolved questions. This demonstrates that the mathematical reasoning capabilities of LLMs are becoming impressive.

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1 Introduction

This project was an attempt to explore the capabilities and limitations of large language models (LLMs) in research-level mathematics. It all started at the [Combinatorial Coworkspace](#) in March 2026, where the three event organizers decided to invite research mathematicians to contribute and discuss benchmark questions using the [ScienceBench](#) platform:

<https://math.sciencebench.ai/benchmarks/benchmarks-in-leipzig>.

This document reports on this compilation and in particular on the *Benchmarks in Leipzig* workshop which took place on May 11–13, 2026, at the [Max Planck Institute for Mathematics in the Sciences](#) in Leipzig, Germany:

<https://www.mis.mpg.de/events/series/benchmarks-in-leipzig>.

Key findings

- The concept of writing exercise-style benchmark questions based on publicly accessible research has reached its limits when it comes to the best-performing available models, see Table 7. For all other models, this approach remains insightful.
- The considered LLMs differ drastically in their performances, see Tables 2 and 6.
- The performance of each model in Table 6 varies drastically between different runs.
- Many of the involved researchers used the ScienceBench chat, possibly indicating that research mathematicians are interested in using AI chat tools when given access to reasoning models in a trusted environment.
- The LLM review of submissions and the comparison against model-generated solutions led to corrections in several submissions, and to three questions being removed from the benchmark. This indicates the potential benefit of AI support in the peer review of mathematical content.

During the question contribution period, April 1–May 15, 2026, **49 research mathematicians** contributed benchmark questions, and 35 of them participated in person in the Benchmarks in Leipzig workshop. During the workshop, the participants combined research exploration via the **ScienceBench chat interface** with structured benchmark submission. The per-user chat has the same project-active models that run on the benchmark, see Table 2 and also Section 3. Within the project phase, 39 of the contributors used it, **accumulating 1,067 turns across 483 chats**. The researchers contributed a total of **100 research-level questions** with unique, well-defined answers. Participants were encouraged to create examples that were based on existing and publicly accessible research but which went beyond what could be presented in a research paper. Unlike for the *First Proof project* by [Abouzaid et al. \(2026\)](#), the goal was not to provide solutions to currently unpublished open problems.

The 100 benchmark questions are presented in Appendix A and belong to the following research areas:

Research area		Research area		Research area	
Algebraic Geometry	36	Commutative Algebra	5	Topology	3
Algebraic Combinatorics	22	Algebraic Statistics	4	Complex Analysis	2
Matroid Theory	18	Homological Algebra	4	Euclidean Geometry	2
Representation Theory	15	Polytope Theory	4	Metric Geometry	1
Enumerative Combinatorics	15	Tropical Geometry	4	Probability Theory	1
Combinatorics	14	Analysis	3	Real Algebraic Geometry	1
Discrete Geometry	10	Arithmetic Geometry	3	Theoretical Computer Science	1
Algebra	7	Knot theory	3		
Graph Theory	5	Number Theory	3		

Table 1: Research areas of the questions, generated using Claude Opus 4.7 and subsequently verified by the authors.

Evaluation

It turned out that most of our questions could be solved by at least one state-of-the-art model, though the differences between models, and between multiple runs of the same model, were significant. The evaluation process for each question consisted of three stages:

Stage 1: During the submission process, the following five *project-active models* (left) and their predecessors (right) each attempted the question in a single run². The following table shows the models’ performances:

Model	Publication date	Solved	Model	Publication date	Solved
GPT-5.5	2026-04-23	44	GPT-5.4	2026-03-17	28
Gemini 3.1 Pro	2026-03-17	15	Gemini 3 Pro	2025-11-18	14
Claude Opus 4.7	2026-04-15	13	Claude Opus 4.6	2026-02-04	14
DeepSeek-V4-Pro	2026-04-24	10	DeepSeek-V3.2	2025-12-01	8
Grok 4.3	2026-05-06	6	Grok 4.20	2026-02-20	5

Table 2: Publication dates and performances of the project-active models (left) and their predecessors (right).

In Stage 1, **41 questions remained unsolved** by all models.

Stage 2: After the workshop on May 11–13, **Surge AI** externally queried each model 20 times on each of the 100 questions. Table 3 summarizes the performances of the models. We say that a model *answered* a question during a run if it produced any answer at all. Because each question was run multiple times per model, we distinguish two metrics: a *correct run* is a single model attempt that produced the correct answer, while a *solved question* is one for which at least one of the 20 attempts was a correct run.

Model	Runs	Answered	Correct runs	Solved questions
GPT-5.5	2000	1389 (69 %)	1043 (52 %)	75 (75 %)
Gemini 3.1 Pro	2000	832 (42 %)	406 (20 %)	40 (40 %)
Claude Opus 4.7	2000	1306 (65 %)	294 (15 %)	44 (44 %)

Table 3: Performances of three models based on 20 external runs per question.

Observe that Claude Opus 4.7 had rather few correct runs compared to the number of solved questions. The reason is that it solved many questions correctly only a few times. For instance, there were 19 questions that it solved in no more than 3 of its 20 attempts. We refer to Section 2.6 for additional performance statistics.

In Stage 2, **20 questions remained unsolved** by all models, and in Stages 1–2, **16 questions remained unsolved**.

Stage 3: All 100 questions were then attempted by GPT-5.5 Pro with Extended Thinking enabled via the ChatGPT website. In addition, **Surge AI** queried Gemini 3.1 Pro Deep Think three times for each question. The following table summarizes the performances of the models:

Model	Runs	Answered	Correct runs	Solved questions
GPT-5.5 Pro (Extended Thinking)	300	300 (100 %)	237 (79 %)	88 (88 %)
Gemini 3.1 Pro Deep Think	300	250 (83 %)	141 (47 %)	56 (56 %)

Table 4: Performances of the two heavy-thinking models across 3 runs each.

Note that the two rows cannot be directly compared: when GPT-5.5 Pro failed to produce an answer to a question at all, we reran it multiple times on the ChatGPT website. On the other hand, we did not rerun Gemini 3.1 Pro Deep Think on the questions it failed to answer, which is why it produced no answer on 14, 17, and 19 of the 100 questions across its three runs, respectively. We refer to Section 2.6 for additional performance statistics.

In Stage 3, **8 questions remained unsolved** by all models, and in Stages 1–3, **2 questions remained unsolved**.

This benchmark sits alongside other recent research-level math evaluations such as FrontierMath Glazer et al. (2024), First Proof Abouzaid et al. (2026), IMProofBench Schmitt et al. (2025), SOOHAK Son et al. (2026), RealMath Zhang et al. (2025), and Riemann-Bench Garre et al. (2026).

²If a model timed out or produced an error, we reran the model up to three times. Further, we originally had Grok 4.20 active instead of Grok 4.3; we reran all questions through Grok 4.3 afterward for an up-to-date comparison.

2 Building the Leipzig Benchmark

The contributors jointly compiled the Leipzig Benchmark between April 1 and May 15, 2026, with the bulk of the questions being submitted during the Benchmarks in Leipzig workshop at the MPI-MiS Leipzig, see Section 4. The three event organizers led the invitation process for the contributions. We aimed to have 35 on-site contributors, and then decided to additionally invite 14 external contributors.

2.1 Submission guidelines

Contributors were invited to submit their questions via the ScienceBench platform, see Section 3. They were given the following guidelines:

- The question must have a unique, well-defined, unguessable answer.
 - Possible answers include, for example, a number, a polynomial, a counting function with parameters, or an expression or formula.
 - The answer to the question should not be a proof, “necessary” or “sufficient” conditions, or any similar derivation.
 - The “unguessable” condition roughly means that a reader allowed to guess the answer multiple times without working out any details should not have a realistic chance of guessing correctly.
- Make the question difficult enough so that only a few project-active models (as presented in the [Introduction](#)) can solve it, if any.
 - During the submission workflow (see Section 2.2), only questions answered correctly by at most three models were accepted for final submission.
- The complexity of the question should not arise solely from a computation.
 - It is allowable to use computer algebra systems while solving the question, but such a computation should not be at the core of the question.
- The model attempts should not fail just because (a) they base their answer on a trivially wrong assumption (for example, misinterpreting a variable called π as the number π), or (b) they have not learned the source material yet. This means, in particular, that the solution to the question should not use results from work that is not publicly accessible.

Comments

- A guiding idea was to create examples that are based on existing research but which go beyond what can be presented in a research paper (for example, a lengthy example that does not fit in an article).
- The goal was not to create open problems with proofs that have not yet been published in the literature.
- The restriction to not solely rely on unpublished work turned out to be a notable caveat. Many contributors would have preferred to submit questions related to their current research.
- The property of the answers not being guessable was sometimes hard to achieve, as multiple questions turned out to have only a very small number of possible answers that could be guessed rather easily.

2.2 Submission workflow

The submission workflow on the ScienceBench platform consisted of three stages:

- Stage W1 The contributor entered a question together with its answer.
- Stage W2 The five project-active models, see Table 2, attempted the question. Their answers were compared to the original answer by an LLM judge. If at most three models answered correctly, the contributor could proceed.
- Stage W3 The contributor entered a worked-out solution to the question that resulted in the already-provided answer, as well as relevant references.

Comments

- The solution and references were never used by the models, and they were also not visible to the other contributors. Their sole role was to make the contributors think through their question and their deduction of the answer clearly.

- In multiple cases, writing the solution triggered a modification of the answer because it revealed previous mistakes in the initial construction.
- In practice, multiple contributors reworked their question to make it harder even when it had already passed the threshold of three or fewer models answering it correctly.

2.3 Post-collection audit

The benchmark questions went through three audit stages:

- Stage A1 All contributors were able to see all questions and the corresponding model answers and solutions. In several cases, this triggered discussions that resulted in fixing minor or even major errors in the originally submitted question/answer.
- Stage A2 After the event ended, each question went through an LLM review. Each submission was reviewed for mathematical typos, inconsistencies, missing content, and possible reasoning errors.
- Stage A3 The questions that remained completely unsolved by all models after the final evaluation in Stage 3 went through another review by the original authors.

Comments

- The AI-assisted review of the Leipzig benchmark at the beginning of Stage A2 flagged 16 potential mathematical issues in the submissions and triggered personal email exchanges with their authors, after which the submissions were updated accordingly. This led to 12 (mostly minor) updated question formulations, and 3 questions were subsequently removed from the benchmark.
- This feedback loop continued during the evaluation rounds in Stages 2 and 3. This resulted in 3 answers being corrected by the authors to match GPT-5.5 Pro’s answer, and 1 additional answer being modified to something different from the model’s answer. *The numbers presented in the introduction already reflect these updates.*
- Such modifications in benchmark problem sets after an LLM review are not unusual. Epoch AI, the company behind the *FrontierMath* project, which comprises mathematics problems of all difficulties, announced on May 11, 2026, that an AI-assisted review of their problems in Tiers 1–4 had flagged fatal errors in about a third of the questions, with a thorough human review still in progress and corrected scores forthcoming [Epoch AI \(2026\)](#).

2.4 Single-attempt model runs

In Stage W2 of the submission workflow, the following model runs were involved, see Table 2 for evaluation:

- Check Before a question was sent to the models to solve, it was checked by GPT-5.5 and Gemini 3.1 Pro against the submission guidelines.
- Runs After passing the check, the question was attempted by the five project-active models. The strongest available version of each model was queried via the API; the exact configurations are listed in Table 5.
- Answer After the models finished, their answer was compared to the original answer using GPT-5.5 and Gemini 3.1 Pro as LLM judges.

Model settings

The API configurations used in Stage W2 to query the five project-active models are presented in the following table:

Model	Effort	Maximum Token Usage
GPT-5.5	none / low / medium / high / xhigh	128k / 128k
Gemini 3.1 Pro	minimal / low / medium / high	64k / 64k
Claude Opus 4.7	low / medium / high / xhigh / max	128k / 128k
DeepSeek-V4-Pro	high / max	128k / 384k
Grok 4.3	none / low / medium / high	[no parameter]

Table 5: API settings used for the project-active models.

The effort used in our API calls is shown in bold among the available effort options for each model. The third column lists the maximum allowed token usage (left value) next to the models’ actual maximum (right value).

Comments

- The complexity of the questions caused many models to error or time out. We then reran these models up to three times on such questions; after three failed attempts, we recorded the result as incorrect.
- Web search, code execution, and computation tools were all disabled for the solve runs. Earlier runs with code execution and computational tools enabled showed that the models started brute-force algorithmic approaches to the questions. This resulted in timeouts and errors and the models in general performed worse than they did when code execution was disabled. One example from the “chain of thought” of GPT-5.5: *I’ll translate your question into the [...] framework, then compute the answer exactly via [...] rather than doing simulation.*
- We chose to run Gemini 3.1 Pro, Claude Opus 4.7, and DeepSeek-V4-Pro with submaximal effort because otherwise these models timed out too frequently. Moreover, the maximal token usage for DeepSeek-V4-Pro is set below its model maximum to match the cap used for the other models.

2.5 Multi-run evaluations

Stages 2 and 3 consisted of multiple multi-run evaluations:

Stage 2 All 100 questions were sent to **Surge AI**, which queried each model 20 times per question.

Stage 3 All 100 questions were then attempted three times each by GPT-5.5 Pro accessed by us through the ChatGPT website. Additionally, **Surge AI** queried Gemini 3.1 Pro Deep Think three times on each question.

The overall performances of the models in Stages 2 and 3 are reported in Tables 3 and 4 in the [Introduction](#), respectively.

2.6 Cross-run and cross-model statistics

Beyond the aggregated statistics in Tables 3 and 4, we provide the following refined counts. We start with the distributions of correct solutions produced during the 20 runs per model in Stage 2.

Model	Number of correct runs (out of 20)					
	0	1 to 4	5 to 9	10 to 14	15 to 19	20
GPT-5.5	25	12	9	7	20	27
Gemini 3.1 Pro	60	12	8	5	10	5
Claude Opus 4.7	56	20	8	10	5	1

Table 6: Per-question consistency distribution across the 20 runs in Stage 2.

The next table shows the analogous distributions of correct solutions in the 3 runs in Stage 3:

Model	Number of correct runs (out of 3)			
	0	1	2	3
GPT-5.5 Pro (Extended Thinking)	12	8	11	69
Gemini 3.1 Pro Deep Think	44	10	7	39

Table 7: Per-question consistency distribution across the 3 runs in Stage 3.

We conclude with a cross-comparison of the performance of the two heavy-thinking models against each other:

		Gemini 3.1 Pro Deep Think	
		Solved	Unsolved
GPT-5.5 Pro (Extended Thinking)	Solved	52	36
	Unsolved	4	8

Table 8: Contingency table cross-comparing the performances of the two models in Stage 3.

3 The ScienceBench platform

The benchmark is hosted on the ScienceBench platform at <https://math.sciencebench.ai>. The platform consists of publicly available benchmarks and several components that are accessible to registered users, which include:

- a three-stage submission system described in Section 2.2,
- a browsing interface that lets every registered user view other contributors' questions, comment on them, attempt the questions, and inspect each model's full answer side by side, and
- a chat system that allows users to chat with all project-active models in parallel.

4 The Benchmarks in Leipzig workshop

The bulk of the benchmark was contributed during the three-day Benchmarks in Leipzig workshop held at the Max Planck Institute for Mathematics in the Sciences from May 11 to 13, 2026. Each day consisted of three phases:

1. joint discussions or presentations related to AI methods and tools in mathematics research;
2. discussion and work in small groups;
3. wrap-up: a showcase of some submissions, summary of progress, and a discussion on common difficulties faced by participants in crafting their questions.

The groups were shuffled each day to encourage interactions and collaborations among researchers specializing in different areas of mathematics. This also allowed participants to exchange experiences gained during earlier sessions.

- Day 1 The workshop started with Christian Stump giving a short introduction to the ScienceBench platform (Section 3) followed by an overview of the submission guidelines and restrictions (Section 2). Afterwards, all participants worked in preassigned groups of two to four persons. Participants in each group came from different scientific backgrounds and had a wide range of prior familiarity with ScienceBench and AI research tools.
- Day 2 The second day started with a presentation on the usage of machine learning methods in mathematics research given by Nupur Jain. This was followed by a discussion led by Bernd Sturmfels on the extent to which AI tools are, can be, and should be used in mathematics research, and on how these tools are shaping the field. Throughout the day, all participants worked on and discussed their submissions, sometimes using the ScienceBench chat system to do so. Towards the end of the day, a few submissions were showcased by their authors and were jointly discussed.
- Day 3 After a brief discussion on the status of the submissions, the remainder of the workshop focused on finalizing drafts, solving and reviewing other participants' submissions, and further exploring the capabilities of the ScienceBench chat system. Some of the newly submitted questions were presented and the models' solutions were compared with the contributors' intended approach. By the end of the workshop, and after the post-collection audit (Section 2.3), the Leipzig Benchmark consisted of 100 submissions, contributed by 35 on-site and 14 off-site contributors, meeting the submission guidelines.

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A The Leipzig Benchmark questions

The 100 Leipzig Benchmark questions are listed below. Each box title carries a label in italics that records the stage at which the question was first answered correctly:

Label	Meaning	Count
<i>solved in Stage 1</i>	correct answer produced by one of the 5 project-active models	59
<i>solved in Stage 2</i>	first solved by one of Surge AI's multi-runs	25
<i>solved in Stage 3</i>	first solved by one of the multi-runs of the two heavy-thinking models	14
<i>remains unsolved</i>	unsolved by all of the above	2

Question 001

solved in Stage 1

Let G be a matrix in $F_3^{4 \times n}$ with rows such that for every vector list of 287 vectors v_1, \dots, v_{287} (there can be repetition) in F_3^4 I can find 287 disjoint subsets X_1, \dots, X_{287} of columns of G such that v_i belongs to the span of the columns of X_i . Let C be a linear code generated by a matrix with the property described above that minimise n . What is the weight enumerator of a code generated by such a G ?

Question 002

solved in Stage 1

Consider the following homogeneous ideal in $\mathbb{C}[z_1, \dots, z_5]$:

$$\begin{aligned} & \langle -z_2 + z_4 - z_5, \\ & z_1^5 z_2^5 + 5z_1^4 z_2^5 z_3 + 10z_1^3 z_2^5 z_3^2 + 10z_1^2 z_2^5 z_3^3 + 5z_1 z_2^5 z_3^4 - z_2^6 z_3^4 + z_2^5 z_3^5 \\ & + 5z_1^5 z_2^4 z_4 + 25z_1^4 z_2^4 z_3 z_4 + 50z_1^3 z_2^4 z_3^2 z_4 + 50z_1^2 z_2^4 z_3^3 z_4 + 25z_1 z_2^4 z_3^4 z_4 + 5z_2^4 z_3^5 z_4 \\ & + 10z_1^5 z_2^3 z_4^2 + 50z_1^4 z_2^3 z_3 z_4^2 + 100z_1^3 z_2^3 z_3^2 z_4^2 + 100z_1^2 z_2^3 z_3^3 z_4^2 + 50z_1 z_2^3 z_3^4 z_4^2 + 10z_2^3 z_3^5 z_4^2 \\ & + 10z_1^5 z_2^2 z_3^3 + 50z_1^4 z_2^2 z_3^3 z_4 + 100z_1^3 z_2^2 z_3^3 z_4^2 + 100z_1^2 z_2^2 z_3^3 z_4^3 + 50z_1 z_2^2 z_3^4 z_4^3 + 10z_2^2 z_3^5 z_4^3 \\ & + 5z_1^5 z_2 z_3^4 + 25z_1^4 z_2 z_3 z_4^4 + 50z_1^3 z_2 z_3^2 z_4^4 + 50z_1^2 z_2 z_3^3 z_4^4 + 25z_1 z_2 z_3^4 z_4^4 + 5z_2 z_3^5 z_4^4 \\ & + z_1^5 z_4^5 + 5z_1^4 z_3 z_4^5 + 10z_1^3 z_3^2 z_4^5 + 10z_1^2 z_3^3 z_4^5 + 5z_1 z_3^4 z_4^5 + z_3^5 z_4^5 \\ & - 6z_2^5 z_3^4 z_5 - 15z_2^4 z_3^4 z_5^2 - 20z_2^3 z_3^4 z_5^3 - 15z_2^2 z_3^4 z_5^4 - 6z_2 z_3^4 z_5^5 - z_3^4 z_5^6 \rangle. \end{aligned}$$

Can you tell me the Picard group of the projective variety cut out by this ideal?

Question 003

solved in Stage 1

What is the last fall degree of the family of polynomials $\{x^{12456789} + y, y^{12456789} + x, x^{503} y^{503}\}$?

Question 004

solved in Stage 1

Consider the ideal $I \subset \mathbb{Q}[x, y]$ generated by 5 polynomials:

$$\begin{aligned} I = \langle & -1 - ax + x^3 - by + y^2, \\ & -c - dx^2 - y + xy - ey^2, \\ & -fx + x^2 - gy + x^2 y - y^2, \\ & -h + x - kx^2 - my - ny^2 + xy^2, \\ & -p - qx - rx^2 + (57y)/13 + y^3 \rangle, \end{aligned}$$

where $(a, b, c, d, e, f, g, h, k, m, n, p, q, r)$ are rational numbers. There is one set of values for $(a, b, c, d, e, f, g, h, k, m, n, p, q, r)$ for which the quotient $\mathbb{Q}[x, y]/I$ has the structure of a 5-dimensional \mathbb{Q} -vector space with basis given by $\{[1], [x], [x^2], [y], [y^2]\}$. Using those values, find $m + n$.

Question 005
solved in Stage 1

Let A be the connected cyclic Nakayama algebra with Kupisch series $[2, 3]$ and let B be the Auslander algebra of A . Let C be the algebra B/I where I is the two-sided ideal given by the socle of the direct sum of all indecomposable projective-injective B -modules (as a submodule of the regular module). What is the sum of all delooping levels of the indecomposable C -modules?

Question 006
solved in Stage 1

For a linear Nakayama algebra A , denote by n_A the number of indecomposable injective modules over A (up to isomorphism) that have codominant dimension at least 2, and denote by m_A the number of simple modules over A (up to isomorphism) with projective dimension 1 that are not 1-regular. Compute a closed-form formula for $\Sigma_A(n_A - m_A)$, where the sum ranges over all linear Nakayama algebras (up to isomorphism) with n simple modules.

Question 007
solved in Stage 1

Let R be the ring of polynomials in 20 variables and let I be the ideal of R consisting of functions vanishing on the set of points in \mathbf{C}^{20} where some 6 coordinates are equal to one another. Finally, let \mathbf{C} be the R -module which is \mathbf{C} as a vector space, upon which the variables all act by 0. What is the dimension of $\text{Tor}_R^2(I, \mathbf{C})$?

Question 008
solved in Stage 1

For a linear Nakayama algebra A arising from a connected quiver on n vertices, denote by n_A the number of simple (left) modules over A (up to isomorphism) that have projective dimension at least 2. Compute a closed-form formula for $\Sigma_A n_A$, where the sum ranges over all linear Nakayama algebras (up to isomorphism) with n simple modules.

Question 009
solved in Stage 1

Let $n \geq 1$ be an integer. For an integer sequence $w = (w_1, \dots, w_{n+1})$ with distinct positive entries and for a permutation $\pi \in S_n$, define the integer sequence $a(w, \pi) = (a_1, \dots, a_n)$ as follows. Let

$$a_1 = \begin{cases} +\max(w_{\pi(1)}, w_{\pi(1)+1}) & \text{if } w_{\pi(1)} < w_{\pi(1)+1}, \\ -\max(w_{\pi(1)}, w_{\pi(1)+1}) & \text{if } w_{\pi(1)} > w_{\pi(1)+1}, \end{cases}$$

and, if $n \geq 2$, let $(a_2, \dots, a_n) = a(w^-, \pi')$ where w^- is obtained from w by deleting $\max(w_{\pi(1)}, w_{\pi(1)+1})$, and where $\pi' \in S_{n-1}$ is the permutation defined by

$$\pi'(i) = \begin{cases} \pi(i+1) & \text{if } \pi(i+1) < \pi(1), \\ \pi(i+1) - 1 & \text{otherwise.} \end{cases}$$

Define the polynomial $F_w(y, t) \in \mathbb{Z}[y, t]$ by

$$F_w(y, t) = \sum_{\pi \in S_n} y^{\#\{a_i \in a(w, \pi) \mid a_i > 0\}} t^{\#\{i \in \{1, \dots, n-1\} \mid a_i < a_{i+1} \text{ where } a(w, \pi) = (a_1, \dots, a_n)\}}.$$

Determine the coefficient of y^3 in $F_w(y, t)$ for $w = (2, 4, 3, 1, 7, 8, 11, 6, 5, 10, 9)$ as polynomial in t .

Question 010
solved in Stage 1

Let $c = (1, 2, \dots, n)$ be the standard n -cycle in the symmetric group S_n . For p satisfying $\gcd(p, n) = 1$ with multiplicative inverse p' so that $pp' = 1 \pmod n$, let $w \in S_n$ be the permutation with $w(i) \equiv i \cdot p' \pmod n$, so that $wc^p w^{-1} = c$.

Let $B : S_n \rightarrow B_n$ be the positive lift of a permutation to the braid group on n strands. For $n = 7$ and $p = 5$, express the pure braid $B(w)B(c)^p B(w)^{-1} B(c)^{-1}$ in terms of the Artin generators of the pure braid group $t_{ij} := \sigma_i \sigma_{i+1} \cdots \sigma_{j-2} \sigma_{j-1}^2 \sigma_{j-2}^{-1} \cdots \sigma_{i+1}^{-1} \sigma_i^{-1}$.

For example, if $n = 4$ and $p = 3$, $w = [3, 2, 1, 4]$ and we should return the list $[(2, 3), (1, 3), (1, 2)]$, representing the pure braid $t_{23}t_{13}t_{12}$.

Question 011
solved in Stage 1

Given lattice paths P and Q from $(0, 0)$ to (m, r) , with P never rising above Q or incident with Q at any point, we define two matroids in terms of P and Q .

For the first matroid, let $P = p_1p_2 \cdots p_n$ and $Q = q_1q_2 \cdots q_n$. If $p_{u_1}, p_{u_2}, \dots, p_{u_r}$ are the north steps of P with $u_1 < u_2 < \cdots < u_r$ and $q_{l_1}, q_{l_2}, \dots, q_{l_r}$ be the north steps of Q with $l_1 < l_2 < \cdots < l_r$. Let N_i be the interval $[l_i, u_i]$ of integers. Let $M[P, Q]$ be the transversal matroid on the ground set $[n]$ that has the presentation (N_1, N_2, \dots, N_r) . A matroid M for which there exists P, Q such that $M = M[P, Q]$ is called a lattice path matroid.

For the second matroid, label the rows of the diagram enclosed by P and Q , from bottom to top, by 1 through r , and the columns, from left to right, by $r + 1$ to $r + m$. For each $i \in [r]$, let the set A_i consist of i along with the labels of all columns that have a square in row i . The rook matroid $R[P, Q]$ is the transversal matroid with the presentation (A_1, \dots, A_r) . A matroid M for which there exists P, Q such that $M = R[P, Q]$ is called a rook matroid.

Let λ/μ be the skew shape enclosed by the region bounded by P and Q ; then λ/μ determines and is determined by P, Q . For $\alpha/\beta = 888888765/76654321$, find the number of lattice path matroids that are isomorphic to the rook matroid defined by α/β .

Question 012
solved in Stage 1

Define a map s from the set of all permutations of length n to that same set as follows.

1. If $n = 1$, then $s(1) = 1$.
2. If $p = LnR$, that is L is the substring on the left of the entry n , then $s(p) = s(L)s(R)n$.

This operation s is often called the stack sorting.

Let $B_{n,k}$ be the set of permutations p of length n for which all of the following hold.

1. The permutation $s(p)$ avoids the patterns 2314 and 3124.
2. The number of descents of p is $k - 1$.

A *block move* is an operation that turns the permutation

$$p = p_1p_2 \cdots p_i p_{i+1} \cdots p_j p_{j+1} \cdots p_k p_{k+1} \cdots p_\ell \cdots p_n$$

into the permutation

$$p' = p_1p_2 \cdots p_i p_{k+1} \cdots p_\ell p_{j+1} \cdots p_k p_{i+1} \cdots p_j p_{\ell+1} \cdots p_n.$$

That is, a block move interchanges the string $p_{i+1} \cdots p_j$ and the string $p_{k+1} \cdots p_\ell$, for some numbers $i < j < k < \ell$. If $p, p' \in S_n$, and t is the smallest integer so that there exists a sequence of t block moves that turn p into p' , then we say that $t = \mathbf{d}(p, p')$ is the *distance* of p and p' .

For a given permutation p , let $D_p(k)$ be the set of all permutations q that are at distance k from p .

How many pairs of permutations (p, π) are there so that

1. both p and π are of length 30, and
2. $p \in B_{30,15}$, and
3. $\pi \in D_p(10)$, that is, π is at distance 10 from p ?

Question 013
solved in Stage 1

Fix two positive integers r and m . Consider the family of subsets $C_i = \{2i + 1, \dots, 2i + r\}$ where i ranges from 0 to $m - 1$. Let M be the matroid of rank r on $n = 2(m - 1) + r$ elements whose non-bases are exactly the m subsets C_0, \dots, C_{m-1} . Using the notation of the paper <https://arxiv.org/pdf/2502.04980> express the unnormalised symmetrized class $[X_{M, \text{sym}}]$ in the basis of the symmetric part of the degree $n - r$ -part of the Chow ring of the permutohedron given by square free monomials in the differences $H_k := L_{n-1-k} - L_{n-k}$ (where k

runs from 1 to $n - 1$ and we use convention $L_0 = 0$). As a formula of r and m please give me the coefficient of the monomial $H_{r-1} \cdot \prod_{k=1}^{n-r-1} H_{n-k}$ in this expression.

Question 014

solved in Stage 1

Let \mathcal{A} be the hyperplane arrangement that consists of the hyperplanes $\{x_i \pm x_j \mid 1 \leq i < j \leq 11\} \cup \{x_i \mid 1 \leq i \leq 11\}$, and let $F_{\mathcal{A}}(y, t)$ be the coarse flag Hilbert-Poincaré series of \mathcal{A} . What is the coefficient of y^7 in $F_{\mathcal{A}}(-y, y)$?

Question 015

solved in Stage 1

Consider an ordered tree on $n + 1$ vertices with root r . We impose the breadth-first left-to-right order $<_T$ on the vertices of T : vertices are ordered first by distance from the root, and within each level from left to right according to the ordering of children. The covering relation in this order is denoted by \prec_T . Denote by $d(u, v)$ the edge-distance between vertices u and v in T . Let $d(T)$ be the maximum of the distances $d(u, v)$ over all pairs $u \prec_T v$ of vertices. Compute a closed-form formula for the number of ordered trees on $n + 1$ vertices ($n \geq 10$) such that $d(T) = n - 3$.

Question 016

solved in Stage 1

Determine the number of permutations of size 20 that avoid the patterns 1324 and 2314 and have exactly 30 inversions.

Question 017

solved in Stage 1

Fix a multiset

$$M = \{1^3, 2^3, 3^4, 4, 5, 6^2, 7^2, 8^4, 9^3, 10, 11^{13}, 12^{100}, 13, 14^2, 15, 16, 17, 18, 19\},$$

where i^k denotes that i appears with multiplicity k . Let $r = (r_1, \dots, r_{19})$ be a color vector, where each element i is assigned r_i colors. For each such r , consider the colored multiset Eulerian polynomial (i.e., the descent generating polynomial over colored multiset permutations of type (M, r)). For which color vectors r is this polynomial palindromic?

Question 018

solved in Stage 1

Let $a(k)$ denote the number of $\{1324, 1342\}$ -avoiding permutations of size $k + 2$ with exactly k inversions. Determine the generating function $\sum_{k \geq 0} a(k)x^k$.

Question 019

solved in Stage 1

Consider a 20 by 20 square grid with lower left corner at $(0,0)$, upper right at $(20,20)$. What is the number of ways to draw 21 monotone grid paths (each having only E or N steps) through the grid which connect $(0,0), \dots, (20,0)$ with $(20,0), \dots, (20,20)$ so that no two paths have overlapping steps, endpoints and so that two paths may cross over at a point but not just touch, the path starting at $(20,0)$ ends at $(20,12)$ and the path starting at $(10,0)$ ends at $(20,0)$?

Question 020

solved in Stage 1

For a Dyck path d of semilength n , consider its bounce path. The “reversing bounce path” of d is the path that arises by reversing d , considering its bounce path, and reversing it again. For example, for $d = ((\))$, its bounce path is $((\))$ and its reverse bounce path is $(\)$. How many Dyck paths are there for $n = 10$, whose bounce path is equal to their reversing bounce path?

Question 021
solved in Stage 1

For a permutation $\pi \in S_n$ and $i \in [n]$, denote by $\pi \setminus i$ the permutation that has the same relative order as the sequence obtained by deleting i from π written in one-line notation. We call π almost decomposable if at least one of the permutations π , $\pi \setminus 1$, $\pi \setminus \pi_1$, $\pi \setminus n$, or $\pi \setminus \pi_n$ is decomposable.

For all $n \geq 10$, find the smallest value of k such that there exists a 1342-avoiding permutation of size n with k inversions that is not almost decomposable.

Question 022
solved in Stage 1

Let Q be the poset on $\{1, 2, 3\} \times \{1, 2, 3, 4\}$ obtained from the product of two chains by deleting exactly the cover relations

$$(2, 1) < (2, 2), \quad (2, 3) < (2, 4), \quad (3, 2) < (3, 3).$$

For each $\sigma \in S_4$, label $(p, j) \in Q$ by $\ell_\sigma(p, j) = p + 3(\sigma(j) - 1)$. If $\pi = (q_1, \dots, q_{12})$ is a linear extension of Q , define

$$\text{des}_{\ell_\sigma}(\pi) = \#\{i \in \{1, \dots, 11\} : \ell_\sigma(q_i) > \ell_\sigma(q_{i+1})\}.$$

Set

$$F_Q(x) = \sum_{\sigma \in S_4} \sum_{\pi \in L(Q)} x^{\text{des}_{\ell_\sigma}(\pi)}.$$

Determine the coefficients γ_i in the expansion

$$F_Q(x) = \sum_i \gamma_i x^i (1+x)^{\deg F_Q - 2i}.$$

Question 023
solved in Stage 1

Let x be a point in the complex Grassmannian $G(k, n)$, and let M be the corresponding matroid. The Chow class of M is defined as the class of the torus orbit closure $(\mathbb{C}^*)^n x$ in the Chow ring of $G(k, n)$.

Expand the Chow class of the lattice path matroid defined by $URUURRUR$ and $RRURRRUUU$ as a linear combination of Schubert cycles s_λ .

Question 024
solved in Stage 1

Consider the following double cover of graphs: G_1 has 5 vertices and edges $\{(1, 2), (1, 3), (2, 3), (2, 3), (2, 4), (3, 5), (4, 5), (4, 5)\}$, and G_2 has 3 vertices and edges $\{(1, 2), (2, 2), (2, 3), (3, 3)\}$. The map on vertices is given by $1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 2, 4 \mapsto 3, 5 \mapsto 3$. Compute the signed graphic matroid of this double cover, and give the set of bases.

Question 025
solved in Stage 1

The circle graph with 8 edges has the property that on the Bergman fan of its graphic matroid the coarse fan structure coincides with the minimal nested set structure.

Count up to isomorphism the number of all simple connected graphs G with 8 edges such that the associated graphic matroid M has the property that it is connected and that on the Bergman fan of M the coarse fan structure coincides with the minimal nested set fan structure.

Question 026
solved in Stage 1

Let M be the lattice path matroid of the skew shape $(5, 5, 5, 5, 5, 5, 5, 5)/(4, 2, 2, 2, 1, 1)$. Compute the Hilbert series of the Chow ring of M .

Question 027
solved in Stage 1

Determine the normalized volume of the non-nesting rook matroid (see arXiv: 2410.00127) defined by $UUURURRRRRUR$ and $RUURRUURRRRUU$.

Question 028
solved in Stage 1

Consider the rational normal curve in real projective space \mathbb{P}^6 , and consider its image in an affine chart. Place 16 points in convex position on the image of the curve in the affine chart, and take their convex hull. How many distinct combinatorial types of polytopes can you get with this construction?

Question 029
solved in Stage 1

Let \mathcal{A} be a hyperplane arrangement of 9 hyperplanes in a 3-dimensional complex vector space, with the hyperplanes labeled by $\{0, 1, \dots, 8\}$. Suppose that the 1-dimensional flats of the arrangement are given by the following sets of hyperplanes: $\{0, 1, 2\}, \{0, 3, 8\}, \{0, 4, 6\}, \{0, 5, 7\}, \{1, 3, 7\}, \{1, 4, 8\}, \{1, 5, 6\}, \{2, 3, 6\}, \{2, 4, 7\}, \{2, 5, 8\}, \{3, 4, 5\}, \{6, 7, 8\}$. Let X be the visible contours compactification of the projectivization of \mathcal{A} , which is naturally embedded in the torus-orbit closure of the point of $Gr(3, 9)$ corresponding to \mathcal{A} . Let N be the normal bundle of X in this torus-orbit closure. Compute $H^1(X, N)$.

Question 030
solved in Stage 1

Consider the log canonical embedding of the moduli space of marked cubic surfaces, that is, marked del Pezzo surfaces of degree 3. What is the dimension of the ambient projective space of this log canonical model? By way of comparison, for the moduli space $M_{0,n}$ that dimension would be $(n-2)! - 1$. What is the analogous number for moduli of marked cubic surfaces?

Question 031
solved in Stage 1

Let $f : Y \rightarrow X$ be a degree 2 cover of smooth, projective, connected curves over the complex numbers. Assume that L is a line bundle on Y such that f^*f_*L is a stable vector bundle of degree zero on Y and such that σ^*L is not isomorphic to L for a generator σ of the Galois group of the covering Y/X . How many line bundles M on Y have the property that there exists an unramified Galois cover $g : Z \rightarrow Y$ of degree three satisfying $g^*f^*f_*L \simeq g^*f^*f_*(L \otimes M)$?

Question 032
solved in Stage 1

Consider the tensor train variety parametrized via the map

$$\Phi : \mathcal{P} \longrightarrow V_{D,d}^- \subseteq \mathbb{P}(\mathbb{C}^{d_1} \otimes \dots \otimes \mathbb{C}^{d_N}), \quad (M^1, \dots, M^{N-1}, [C]) \longmapsto [T]$$

with

$$\mathcal{P} := \prod_{r=1}^{N-1} \text{St}(D_r, n_r) \times \mathbb{P}^{\mathbb{C}^{D_{N-1} \times d_N}}$$

and $\mathbb{P}^{\mathbb{C}^{D_{N-1} \times d_N}}$ is the projectivization of the space of full rank $D_{N-1} \times d_N$ complex matrices. Using this parametrization, compute the degree of the tensor train variety with $D = (1, 2, 2, 2, 1)$ and $d = (3, 3, 3, 3)$. Do not use Chern classes.

Question 033
solved in Stage 1

Let λ be the partition $(2, 2, 2)$. Let $a = \{t^6, t^5, t^4, t^3, t^2, t\}$ such that, in the notation of this paper <https://arxiv.org/abs/0902.1321v3>. Let $T \in \text{SYT}(\lambda/\mu; a)$ be the tableaux with filling $t^6, t^5, t^4, t^3, t^2, t$ read from top to bottom, left to right.

In Theorem 4.2 in this paper <https://arxiv.org/abs/0902.1321v3> they say that assuming there exists a filling of the tableaux $(2, 2, 2)$ with numbers $\omega_1, \dots, \omega_6$ from left to right and top to bottom such that the determinant of the Jacobian J defined in Theorem 4.2 is non-zero then there exists a linear series in the fiber of $(z + t^6)(z + t^5)(z + t^4)(z + t^3)(z + t^2)(z + t)$ of the Wronski map with certain leading terms of the Plücker coordinates specified in the theorem if and only if the ω_i satisfy certain equations.

Can you find numbers $\omega_1, \dots, \omega_6$ such that the theorem applies that is in the case where there exists a linear series in the fiber of $(z + t^6)(z + t^5)(z + t^4)(z + t^3)(z + t^2)(z + t)$ of the Wronski map. There should be a unique answer in this case. What is the sum $\omega_1 + \dots + \omega_6$?

Question 034
solved in Stage 1

Let E be an elliptic curve over the field \mathbb{C}_p without complex multiplication. We assume that E has good supersingular reduction. We define X as the perfectoid cover of $E \times E$ which is the inverse limit along the infinite tower $p : E \times E \rightarrow E \times E$ given by p -multiplication. What is the rank of the Néron-Severi group of X over $\mathbb{Z}[1/p]$?

Question 035
solved in Stage 1

Let X be the variety of symmetric 6×6 matrices of rank at most 4, viewed as a subvariety in a projective space \mathbb{P}^{20} of dimension 20. We are interested in the coisotropic hypersurface $\text{Ch}_3(X)$ which consists of all linear spaces of dimension 5 in \mathbb{P}^{20} which fail to intersect X transversally. What is the degree of this hypersurface in Plücker coordinates on the Grassmannian $\text{Gr}(5, \mathbb{P}^{20})$?

Question 036
solved in Stage 1

Let X be the Segre embedding of $(\mathbb{P}^1)^{\times 3}$ in \mathbb{P}^7 , and consider its affine cone CX in \mathbb{C}^8 . Fix coordinates (x_1, \dots, x_8) for \mathbb{C}^8 . What is the number of complex critical points of the function $f(x) = 2(x_1 - u_1)^2 + \sum_{i=2}^8 (x_i - u_i)^2$ from a generic point $u = (u_1, \dots, u_8) \in \mathbb{C}^8$ restricted to CX ?

Question 037
solved in Stage 1

Consider the cubic threefold X in $\mathbb{R}\mathbb{P}^4$ defined by the following equation:

$$x_0x_1x_2 + x_1x_2x_3 + x_2x_3x_4 + x_0x_3x_4 + x_0x_1x_4 = 0$$

Let Y be the union of the two-dimensional linear spaces contained in X . How many connected components does $X \setminus Y$ have?

Question 038
solved in Stage 1

Let $[L]$ be a general point of the Grassmannian $\text{Gr}(6, 17)$, and let X denote the visible contour compactification of the corresponding hyperplane arrangement. Then X is naturally embedded in the torus-orbit closure of $[L]$. Let E denote the conormal bundle of X in this toric variety. Compute the degree of $c_{\dim X}(E)$.

Question 039
solved in Stage 1

Let Z be the free graded commutative algebra with one generator in each degree, except in degrees 1, 4, and 6. For a natural number k , let X_k be the projective toric variety parametrized by the degree k monomials in Z (inside $\mathbb{P}(Z_k)$). What is the largest k such that X_k is normal?

Question 040
solved in Stage 1

Fix seven general points in the real projective plane. Consider the 21 lines spanned by any two points, the 21 conics spanned by any five points, and the 7 cubics through all points that pass doubly through one of the seven points. These $49 = 21 + 21 + 7$ curves divide the real projective plane into regions, called polypols, each of which has a unique canonical differential form. How many of these canonical forms are linearly independent?

Question 041
solved in Stage 1

Let Σ be a complete convex fan in \mathbb{R}^n and X its toric variety with torus $T = (\mathbb{C}^*)^n$ acting on X . Fix a point $x^* \in X$ such that the orbit of x^* sits dense in X . Let $Z_C \subset X$ be the torus orbit closure corresponding to the d -dimensional cone $C \in \Sigma$. We call X equivariantly simple at Z_C iff for every $L \subset \mathbb{C}^n$ linear space and every sequence $(t_n)_{n \geq 0} \subset T$ with limit $t = \lim_{n \rightarrow \infty} (t_n \cdot x^*) \in (Z_C)^\circ$ (where $(Z_C)^\circ$ is the open subset of (Z_C) with no zero coordinates), the limit $\lim_{t \rightarrow \infty} (t_n \cdot L) \in \text{Gr}(\dim(L), \mathbb{C}^n)$ depends only on t , where $t_n \cdot L$ is the inverse coordinate-wise action of T on L . Let Σ be the complete fan normal to the polytope whose vertex coordinates are $(1, 2, 3), (0, 2, 3), (1, 3, 2), (0, 3, 2), (1, 0, 2), (1, 3, 1)$. We realise $X \subseteq \mathbb{P}^7$ as the closure of the image of the map $(\mathbb{C}^*)^3 \rightarrow \mathbb{P}^7, (t_0, t_1, t_2) \mapsto (t_1^2 t_2^3 : t_1^3 t_2^2 : t_0 t_2^2 : t_0 t_1 t_2^2 : t_0 t_1^2 t_2^2 : t_0 t_1^3 t_2^2 : t_0 t_1^2 t_2 : t_0 t_1^3 t_2)$ and let $T = (\mathbb{C}^*)^3$

act on X via the induced action of the multiplication in the domain of this parametrization. A subdivision of a fan consists of picking a maximal cone of the fan and a hyperplane and replacing the cone by the two cones obtained by intersecting the initial cone with the two half-spaces of the hyperplane. What is the minimal number of consecutive subdivisions needed to make X equivariantly simple?

Question 042
solved in Stage 1

Let $X \subseteq \mathbb{P} := \mathbb{P}^3 \times \mathbb{P}^3$ be the image of the map $(x, y, z, w) \mapsto ([xy : z : w : 1], [zw : x : y : 1])$. In the following, we denote by $\mathbb{G}(k, n)$ the Grassmannian of k -dimensional projective subspaces of \mathbb{P}^n . Consider the product of Grassmannians $\mathbb{G} := \mathbb{G}(1, 3) \times \mathbb{G}(1, 3)$. For a generic $L = L_1 \times L_2 \in \mathbb{G}$, since $\dim(L) = 2 = \text{codim}(X)$, we expect that $X \cap L$ is either empty or a 0-dimensional reduced scheme of \mathbb{P} . Let $I_\alpha \subseteq \mathbb{G}$ be the locus of products of linear spaces $L \in \mathbb{G}$ such that the intersection $X \cap L$ is nontransversal (in particular, a 0-dimensional nonreduced set). The locus I_α is a hypersurface in \mathbb{G} , in particular, it has a multidegree $\beta = (\beta_1, \beta_2)$, where β_i is the degree in the Plücker coordinates of the i -th Grassmannian $\mathbb{G}(1, 3)$. What is the value of $\beta_1 + \beta_2$?

Question 043
solved in Stage 1

Let $X = \text{Seg}(\mathbb{P}^3 \times \mathbb{P}^3) \subset \mathbb{P}(\mathbb{C}^4 \otimes \mathbb{C}^4)$ be the Segre variety of rank-1 matrices, $\sigma_2(X)$ its 2nd secant variety (consisting of rank-2 matrices), and

$$u = e_1^{\otimes 2} + e_2^{\otimes 2} + \frac{1}{100} e_3^{\otimes 2} + \frac{1}{100} e_4^{\otimes 2} \in \mathbb{R}^{4 \times 4}.$$

Consider the polynomial optimization problem

$$\min_{\substack{a^{(k)}, b^{(k)} \in \mathbb{R}^4 \\ k=1,2}} f(a, b) := \sum_{i,j=1}^4 \left(u_{ij} - \sum_{k=1}^2 a_i^{(k)} b_j^{(k)} \right)^2 \quad (1)$$

$$\text{subject to } f(a, b) \leq 1/50. \quad (2)$$

Use the standard Moment-SOS hierarchy for this Polynomial Optimization Problem (POP), where the d -th level optimizes over polynomials of degree at most $2d$. If the hierarchy has finite convergence, set $s=1$. Otherwise, set $s=-1$. Compute $(s \cdot f_2^*, s \cdot f_3^*, s \cdot f_4^*)$, where f_d^* denotes the optimal value of the d -th level of the hierarchy.

Question 044
solved in Stage 1

Let K be an algebraically closed and complete extension field of \mathbb{Q}_p and let X be a smooth, proper, connected curve over K of genus g . We look at the associated diamond X^\diamond defined by Peter Scholze. Let \mathcal{O} be its structure sheaf. X^\diamond carries an étale topology which we denote by et and a quasi-pro-étale topology which we denote by $qproet$.

Look at the inclusion $Ext_{et}^1(\mathcal{O}, \mathcal{O}) \rightarrow Ext_{qproet}^1(\mathcal{O}, \mathcal{O})$ given by self-extensions of the structure sheaf on X^\diamond with respect to these two topologies. Compute the dimension of the cokernel of this inclusion as a K -vector space. If it is not finite dimensional, give back ∞ .

Question 045
solved in Stage 1

Let $N = 9$ and λ be the partition $(3, 2, 1)$ and $\mu = (3, 1, 0)$. Let $a = (t^2, t)$ such that $a^+ = a \cup \{0, 0, 0, 0, \infty, \infty, \infty\}$, in the notation of this paper: [arXiv:0902.1321v3](https://arxiv.org/abs/0902.1321v3). Let $T \in \text{SYT}(\lambda/\mu; a)$ be the tableau with filling of the skew shape with t in the bottom row and t^2 in the cell above and to the right of this cell. In the language of the just mentioned paper the tableau is a diagonally increasing skew tableau.

According to Theorem 4.2 in the same paper there exists a linear series x , with certain properties. Let f be the map $\mathbb{P}^1 \rightarrow \mathbb{P}^2$ corresponding to x . Consider $(f, \{t^2, t\})$ as an element in the moduli space of stable maps, with marked points at t^2, t .

In the limit $t \rightarrow 0$ in the moduli space of stable maps this becomes a degree 4 plane curve with a degree zero component X attached at c_∞ that has one marked point on it at c_t and one at c_0 , corresponding to the marked points t, t^2 , respectively. There is a degree 1 component attached to X at c_* .

Assume that X is parametrized such that c_∞ is at $c = \infty$, c_0 is at $c = 0$, and c_t is at $c = 1$. At what value of c is the node c_* ?

Question 046

solved in Stage 1

Imagine you are an observer agent living in a universe with a smooth continuum of visible angular directions. Your visible sky is modeled as the complex projective line $\mathbb{P}^1(\mathbb{C})$, equipped with normalized Fubini-Study probability measure σ . You as an observer are not told whether this continuum is fundamental, or whether it is the low-energy rendering/simulation of a hidden finite algebraic substrate. If you are a capable observer to figure out whether you live in a simulation or not, you should be able to infer, from a small statistical defect in the visible angular distribution together with a few invariant-theoretic fingerprints, whether the continuum hypothesis is best explained by an underlying quotient substrate. You not given the hidden group, the primitive invariant, or the perturbation strength. These must be reconstructed from the algebraic data and then used in an asymptotic decision problem.

Let $U = \mathbb{C}^2$, with homogeneous coordinates (u, v) , and let σ be the normalized Fubini-Study probability measure on $\mathbb{P}^1(\mathbb{C})$. Under the alternative hypothesis, a hidden computational substrate is assumed to have a non-cyclic finite $\mathrm{SL}_2(\mathbb{C})$ -quotient singularity $X_\Gamma = \mathrm{Spec} \mathbb{C}[u, v]^\Gamma$, for some unknown finite subgroup $\Gamma \subset \mathrm{SL}_2(\mathbb{C})$. The visible angular sky is $\mathbb{P}^1(\mathbb{C})$, and the local statistical defect is generated by the lowest primitive invariant of the quotient singularity.

The substrate is known only through the following side information.

1. The minimal resolution of X_Γ has exceptional intersection graph E_8 .
2. Equivalently, $\mathbb{C}[u, v]^\Gamma$ has primitive homogeneous generators of degrees 12, 20, 30, satisfying one weighted-homogeneous relation of degree 60.
3. The degree-12 primitive invariant is normalized as follows. It lies in the one-parameter family

$$F_a(u, v) = u^{11}v + a u^6 v^6 - uv^{11},$$

and $a \in \mathbb{C}$ is chosen so that, if

$$H_a = \mathrm{Hess}(F_a) = \det \begin{pmatrix} \frac{\partial^2 F_a}{\partial u^2} & \frac{\partial^2 F_a}{\partial u \partial v} \\ \frac{\partial^2 F_a}{\partial v \partial u} & \frac{\partial^2 F_a}{\partial v^2} \end{pmatrix},$$

and

$$T_a = \mathrm{Jac}(F_a, H_a) = \det \begin{pmatrix} \frac{\partial F_a}{\partial u} & \frac{\partial F_a}{\partial v} \\ \frac{\partial H_a}{\partial u} & \frac{\partial H_a}{\partial v} \end{pmatrix},$$

then the three degree-60 binary forms F_a^5, H_a^3, T_a^2 , are linearly dependent over \mathbb{C} , with all three coefficients in the linear dependence nonzero. This is the intrinsic syzygy condition seen by the observer.

For such an admissible value of a , define the projectively well-defined nonnegative function

$$Q([u : v]) = \frac{|F_a(u, v)|^2}{(|u|^2 + |v|^2)^{12}}, \quad [u : v] \in \mathbb{P}^1(\mathbb{C}),$$

and its mean-zero normalization

$$\rho([u : v]) = \frac{Q([u : v])}{\int_{\mathbb{P}^1} Q d\sigma} - 1.$$

For sufficiently small ε , define a probability measure P_ε on $\mathbb{P}^1(\mathbb{C})$ by

$$\frac{dP_\varepsilon}{d\sigma} = 1 + \varepsilon \rho.$$

The null hypothesis is that the visible sky is statistically indistinguishable from the continuum model, $H_0 : \varepsilon = 0$. The alternative hypothesis is that the observed sky is the continuum rendering of the hidden quotient substrate, $H_1 : \varepsilon = \varepsilon_0 > 0$. You as an observer see n independent samples from P_ε , where $\varepsilon \in \{0, \varepsilon_0\}$. You as an observer scientist use the Neyman/Pearson likelihood-ratio test for

$$H_0 : \varepsilon = 0 \quad \text{against} \quad H_1 : \varepsilon = \varepsilon_0 > 0.$$

In the local regime $\varepsilon_0 = \frac{h}{\sqrt{n}}$, determine the exact constant C such that, when $h = C$, the asymptotic Type-I and Type-II errors are both $\alpha = \beta = \Phi(-1)$.

Your answer must include the following five items:

1. the hidden group Γ , up to conjugacy in $\mathrm{SL}_2(\mathbb{C})$
2. the admissible value or values of a
3. the exact Fisher information $\mathcal{I} = \int_{\mathbb{P}^1} \rho^2 d\sigma$
4. the exact threshold constant $C = \frac{2}{\sqrt{\mathcal{I}}}$
5. the limiting log-Bayes factor, in favor of the hidden quotient-substrate sky over the continuum sky, assuming equal prior odds.

Question 047
solved in Stage 1

Let X be the variety in $(\mathbb{C}^*)^5$ cut out by $x_3x_4 + x_1 - 1, x_5 + x_2x_3 - 1, x_4x_5 + x_2x_3x_4 + x_1x_2 - 1$. Consider the form

$$\Omega = \frac{dx_1 \wedge dx_2}{x_1x_2} + \frac{dx_1 \wedge dx_3}{x_1x_3} - \frac{dx_1 \wedge dx_5}{x_1x_5} + \frac{dx_2 \wedge dx_3}{x_2x_3} + \frac{dx_2 \wedge dx_4}{x_2x_4} + \frac{dx_3 \wedge dx_4}{x_3x_4} + \frac{dx_3 \wedge dx_5}{x_3x_5} + \frac{dx_4 \wedge dx_5}{x_4x_5}.$$

Let $\mathrm{Crit}_X(s)$ be the critical points on X of the multivalued function $\log(x_1^{s_1} \cdots x_5^{s_5})$. Write a closed form in terms of s for

$$\sum_{x(s) \in \mathrm{Crit}_X(s)} \Omega(x(s)).$$

Question 048
solved in Stage 1

Suppose that I have a graph composed of a chain of 10 cycles of length 20. Each cycle is joined by an edge to the cycle before and after, with the attachment points diametrically opposed to each other. Let v be the vertex on the last cycle diametrically opposed to the attachment point with the previous chain.

Determine the number of degree 4 effective and v -reduced divisors D such that $D + 5v$ has Baker–Norine rank at least 3.

Question 049
solved in Stage 1

Let $\mathrm{Gr}(2, n)$ be the Grassmannian of lines in \mathbb{C}^n with its Plücker embedding. Compute the combinatorial rank $\mathrm{cr}(\mathrm{Gr}(2, 4), D)$, where D is the union of all Plücker hyperplane sections.

Question 050
solved in Stage 1

Consider the Segre-Veronese embedding of $\mathbb{P} = (\mathbb{P}^1)^{\times 3}$ associated with the line bundle $\mathcal{L} = \mathcal{O}_{\mathbb{P}}(124534, 2345729, 87685764)$. Let $\mathcal{P}^{100}(\mathcal{L})$ denote the jet bundle of order 100 of \mathcal{L} . For all $i \geq 0$, let $c_i(\mathcal{P}^{100}(\mathcal{L}))$ denote the i -th Chern class of $\mathcal{P}^{100}(\mathcal{L})$. Compute the value of

$$\sum_{i=0}^3 \deg(c_i(\mathcal{P}^{100}(\mathcal{L})))$$

Question 051
solved in Stage 1

Let $f_t : \mathbb{P}^1 \rightarrow \mathbb{P}^2$ be a family of maps of degree 3 parametrized by t . Consider (f_t, p_1, p_2, p_3) as a family of stable maps with 3 marked points where the marked points p_1, p_2, p_3 are on the flexes of f_t . In the case where the 3 flexes collide as $t \rightarrow 0$ and the domain of the map splits into a degree 2 component C , a degree 1 component E_1 , and a degree 0 component E_0 . Assume that C is connected to E_0 and E_1 is connected to E_0 . Assume that E_0 is parametrized by c such that the node between C and E_0 is at $c = \infty$ and there is at least one marked point at $c = 0$ and at least one at $c = 1$ what is the highest possible value of c of the location of the node between E_0 and E_1 ?

Question 052
solved in Stage 1

Consider any Dyck path D in the grid \mathbb{Z}^2 that stays above the diagonal $(1, 0)$ to $(n + 1, n)$. Take P as the Minkowski sum of all simplices $\Delta_{ij} = \text{Newt}(1 + x_i + x_i x_{i+1} + \dots + x_i \cdots x_j)$ for all (i, j) on or below D . How many facets does P have?

Question 053
solved in Stage 1

Suppose I have two functions $f_1 : \mathbb{R}^d \rightarrow \mathbb{R}^n$, and $f_2 : \mathbb{R}^n \rightarrow \mathbb{R}$. The functions are of the form $f_i(x) = \max(A_i x, B_i x)$. Here, A_1, B_1 are $n \times d$ matrices with real entries, and A_2, B_2 are $1 \times n$ matrices with non-negative real entries. The composition $f(x) = f_2(f_1(x))$ is convex, continuous, piecewise-linear and positively homogeneous. Its dual polytope is denoted P_f . For $d = 2$, $n = 1003$, determine $\max\{\#\text{ vertices } P_f : f \text{ is as described above}\}$.

Question 054
solved in Stage 1

We consider the space X of all closed C^2 -submanifolds $S \subset (\mathbb{R}/\mathbb{Z})^2$ of dimension 1 such that for every $a \in \mathbb{Z}^2$ with positive entries the map $\pi_a : S \rightarrow \mathbb{R}/\mathbb{Z}$, $x \mapsto \langle x, a \rangle$ is a covering map and the map

$$\text{tr}_a : \mathbb{R}/\mathbb{Z} \rightarrow (\mathbb{R}/\mathbb{Z})^2, y \mapsto \sum_{x \in \pi_a^{-1}(y)} x$$

is affine linear with slope $(11, 5)$. We equip X with the Hausdorff distance. What is the Lebesgue covering dimension of X ?

Question 055
solved in Stage 1

Let $G(a_1, a_2, \dots, a_n; z)$ be a Goncharov polylogarithm, such that for complex numbers a_1, a_2, \dots, a_n , and z , are defined recursively by:

$$G(; z) = 1$$

$$G(a_1, a_2, \dots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \dots, a_n; t)$$

with the common shuffle regularization: $G(0; z) = \log(z)$; Another examples are $G(1; z) = \log(1 - z)$, and $G(x; z) = \log(1 - z/x)$ where $\log(z)$ is the natural logarithm;

Now consider the analytic continuation of polylogarithms: $G(x; y) = G(y; x) + i\pi + G(0; y) - G(0; x)$ which is done with a fixed branch choice. Note that this has the same information as the more familiar formula:

$$\log(x - y) = i\pi + \log(y - x)$$

With that same branch choice, obtain a formula for $G(x, 1; y)$ in terms of polylogs where y is only allowed to be on the right of the semicolon when the numbers to the left of the semicolon are 0 or 1, and where x is allowed to be in the right of the semicolon when the numbers to the left of the semicolon are either of 0, 1 or y .

Question 056
solved in Stage 1

Consider the following matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

We denote the i -th column of A as α_i , and we write $A = [\alpha_1, \dots, \alpha_{16}]$. Now, for a fixed choice of $W = (w_1, \dots, w_{16}) \in (\mathbb{C}^*)^{16}$ consider the polynomial map

$$\Phi_W : (\mathbb{C}^*)^5 \rightarrow \mathbb{C}^{16}$$

given by $\Phi_W(t) = (w_1 t^{\alpha_1}, \dots, w_{16} t^{\alpha_{16}})$. What is the expected maximum likelihood degree of the Zariski closure of the image of this map for the choice $W = (1, 1, 1, 1, 1, -1, -1, 1, 4, -2, -2, 1, c_1, c_2, c_3, c_4)$, where c_1, c_2, c_3, c_4 are generic complex numbers in the torus?

Question 057
solved in Stage 1

What is the maximum likelihood degree of the Grassmannian of lines $\text{Gr}(2, n)$ in its Plücker embedding in $\mathbb{P}^{\binom{n}{2}-1}$?

Question 058
solved in Stage 1

Let G_1, \dots, G_N be independent standard Gaussian vectors in \mathbb{R}^2 , and set

$$S_N := \frac{1}{N} \sum_{j=1}^N G_j G_j^\top.$$

For N large enough, define

$$Y_j^{(N)} := S_N^{-1/2} G_j, \quad j = 1, \dots, N.$$

Set

$$u_1 = (1, 0), \quad u_2 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \quad u_3 = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right).$$

For $u \in S^1$, define

$$Q_N(u) := \frac{1}{N} \sum_{j=1}^N \left| \langle Y_j^{(N)}, u \rangle \right|^6.$$

Finally, let

$$B := \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}.$$

Determine the exact value of

$$\lim_{N \rightarrow \infty} \frac{1}{N^{1/3}} \log \mathbb{P} \left(Q_N(u_1) \geq 27, Q_N(u_2) \geq 27, Q_N(u_3) \geq 27, N^{1/3}(S_N - I_2) \succeq B \right),$$

where \succeq denotes the Loewner order on symmetric matrices.

Question 059
solved in Stage 1

Consider the definition of Grassmann Distance Degrees of subvarieties of a Grassmannian $\text{Gr}(2, n)$ as in <https://arxiv.org/pdf/2601.22843>.

Given a partition A_1, \dots, A_r of $[n]$ with $|A_1| \geq |A_2| \geq \dots \geq |A_r|$, we can define an associated rank-two matroid on n elements by declaring $e \neq f$ to be a non-basis if and only if there exists j such that $e, f \in A_j$. Denote this by $M_{(|A_1|, \dots, |A_r|)}$ and set $m_i = |A_i|$. Such a matroid is always realized by the $2 \times n$ matrix whose columns are partitioned into r blocks of sizes m_1, \dots, m_r (ordered such that the first m_1 columns form the first block and so on) and such that two columns are linearly independent if and only if they belong to different blocks. The rowspans of such matrices form a subvariety of the Grassmannian $\text{Gr}(2, n)$.

What is the Grassmann Distance Degree of the subvariety corresponding to a partition of sizes $(m_1, \dots, m_5) = (2, 2, 2, 2, 2)$ in $\text{Gr}(2, 10)$?

Question 060
solved in Stage 2

Call a finite dimensional algebra A self-orthogonal if every indecomposable A -module M is self-orthogonal meaning $\text{Ext}_A^i(M, M) = 0$ for all $i > 0$. Consider the class of all connected cyclic Nakayama algebras given by quiver and admissible relations over a field K with 8 simple modules that have dominant dimension at least two and are self-orthogonal. What is the sum of the vector space dimensions of those algebras?

Question 061
solved in Stage 2

 Consider the following symmetric tensor $T \in \mathbb{C}^{6 \times 6 \times 6}$:

$$\begin{aligned}
 T_{::,1} &= \begin{pmatrix} 25 & 20 & 17 & 15 & -16 & -6 \\ 20 & -4 & 2 & -13 & 30 & 27 \\ 17 & 2 & 59 & -28 & 54 & 60 \\ 15 & -13 & -28 & 19 & -2 & 0 \\ -16 & 30 & 54 & -2 & 8 & 15 \\ -6 & 27 & 60 & 0 & 15 & 54 \end{pmatrix}, & T_{::,2} &= \begin{pmatrix} 20 & -4 & 2 & -13 & 30 & 27 \\ -4 & 28 & 23 & 35 & -36 & 0 \\ 2 & 23 & 23 & 26 & -18 & -9 \\ -13 & 35 & 26 & 55 & -51 & 3 \\ 30 & -36 & -18 & -51 & 54 & 3 \\ 27 & 0 & -9 & 3 & 3 & -12 \end{pmatrix}, \\
 T_{::,3} &= \begin{pmatrix} 17 & 2 & 59 & -28 & 54 & 60 \\ 2 & 23 & 23 & 26 & -18 & -9 \\ 59 & 23 & 27 & 32 & -13 & 18 \\ -28 & 26 & 32 & 62 & -42 & -15 \\ 54 & -18 & -13 & -42 & 49 & 21 \\ 60 & -9 & 18 & -15 & 21 & 12 \end{pmatrix}, & T_{::,4} &= \begin{pmatrix} 15 & -13 & -28 & 19 & -2 & 0 \\ -13 & 35 & 26 & 55 & -51 & 3 \\ -28 & 26 & 32 & 62 & -42 & -15 \\ 19 & 55 & 62 & 72 & -43 & 39 \\ -2 & -51 & -42 & -43 & 37 & -27 \\ 0 & 3 & -15 & 39 & -27 & -21 \end{pmatrix}, \\
 T_{::,5} &= \begin{pmatrix} -16 & 30 & 54 & -2 & 8 & 15 \\ 30 & -36 & -18 & -51 & 54 & 3 \\ 54 & -18 & -13 & -42 & 49 & 21 \\ -2 & -51 & -42 & -43 & 37 & -27 \\ 8 & 54 & 49 & 37 & -29 & 33 \\ 15 & 3 & 21 & -27 & 33 & 15 \end{pmatrix}, & T_{::,6} &= \begin{pmatrix} -6 & 27 & 60 & 0 & 15 & 54 \\ 27 & 0 & -9 & 3 & 3 & -12 \\ 60 & -9 & 18 & -15 & 21 & 12 \\ 0 & 3 & -15 & 39 & -27 & -21 \\ 15 & 3 & 21 & -27 & 33 & 15 \\ 54 & -12 & 12 & -21 & 15 & 9 \end{pmatrix}.
 \end{aligned}$$

Determine the rank r and the unique minimum rank decomposition of T , so that $T = \sum_{i=1}^r a_i^{\otimes 3}$, where $a_1, \dots, a_r \in \mathbb{C}^6$. As your answer, submit the tuple $(r, \sum_{i=1}^r |a_{i,3}|^2)$, consisting of the rank r and the “checksum” of 3rd coordinates $a_{i,3}$ of the vectors in the minimum rank decomposition.

Question 062
solved in Stage 2

Let $p > 5$ be a prime and \mathbb{Z}_p the p -adic integers. Let $G = \mathrm{GL}_{12}$ over \mathbb{Z}_p (Dynkin type A_{11}). Let $\check{\mathbb{Z}}_p$ be the integers of the completed unramified closure of \mathbb{Q}_p . For $r > 0$ regard $G_r := G(\check{\mathbb{Z}}_p/p^r)$ as (the $k := \overline{\mathbb{F}}_p$ points of) a perfect algebraic group (of perfectly finite type) over the residue field \mathbb{F}_p by using the positive loop functor (also called jet scheme) construction. Let $G_r^1 := \ker(G_r \rightarrow G_1)$. Denote by F the geometric Frobenius on G_r . Let T be an elliptic unramified maximal torus of Coxeter type of G (defined over \mathbb{Z}_p) and let U be the unipotent radical of a Borel subgroup containing T (only defined over $\check{\mathbb{Z}}_p$). We have the corresponding subgroups T_r^1, U_r^1 of G_r^1 , and T_r^1 is F -stable.

The k -variety $X_r = \{g \in G_r^1 : g^{-1}F(g) \in U_r^1\}$ has an action of $(T_r^1)^F$ by $t : x \mapsto xt$. Let $\phi : (T_r^1)^F \rightarrow \Lambda^\times$ be a character, where $\Lambda = \overline{\mathbb{Q}}_\ell$ and $\ell \neq p$ is a prime.

Let T^1 be the preimage in $G(\check{\mathbb{Z}}_p)$ of $T_r^1 \subseteq G_r^1$. Let ψ be any lift to $T(\mathbb{Q}_p)$ of the inflation of ϕ to $T^1(\mathbb{Z}_p)$. Assume ψ has a Howe factorization in the sense of Kaletha (see section 3.6 of Kaletha’s paper “Regular supercuspidal representations”), such that $\psi = \psi_{-1} \cdot \psi_0 \cdot \psi_1$ with ψ_i of depth r_i with $r_{-1} := 0 < r_0 < r_1 \leq r$ and the root systems of corresponding twisted Levi sequence is $G^{-1} = T \subsetneq G^0 \subsetneq G^1 = G$, with G^0 of (absolute) type $A_5 \times A_5$. (Note that ψ_{-1} will not affect this question, as $\psi|_{T^1(\mathbb{Z}_p)}$ does not depend on the depth zero character ψ_{-1} .)

Question: assume $r = 20, r_0 = 12, r_1 = 15$. Determine the smallest $i \geq 0$ for which the ϕ -weight part of the étale cohomology group with compact support $H_c^i(X_r, \Lambda)[\phi]$ is non-zero.

Question 063
solved in Stage 2

Let A be the connected linear Nakayama algebra with Kupisch series $[5, 5, 4, 4, 3, 3, 3, 2, 1]$ and let T denote the trivial extension algebra of A by $D(A)$. What is the sum of all the Ω -periods of the simple T -modules?

Question 064
solved in Stage 2

For a connected Nakayama algebra A with n simple modules with associated Kupisch series $c(A) = [c_0, \dots, c_{n-1}]$, define ΣA as the sum of all entries of the Kupisch series. Let L_n be the set of all connected Nakayama algebras A (linear and cyclic) with n simple modules that are higher Auslander algebras (meaning the global dimension and the dominant dimension of A are finite and coincide) and for which the unique cycle in the resolution quiver of A consists of exactly 1 vertex. Compute $\sum_{A \in L_n} \Sigma A$ for $n = 20$.

Question 065
solved in Stage 2

Consider the two posets P and Q given by the cover relations

$$\begin{array}{ll}
 P : & Q : \\
 0 < a_1, a_2, a_3 & 0 < a_1, a_2, a_3 \\
 a_1 < b_1, b_2 & a_1 < b_1, b_2, b_3 \\
 a_2 < b_2, b_3 & a_2 < b_2, b_3 \\
 a_3 < b_3, b_1 & a_3 < b_1 \\
 b_1 < c_1, c_2 & b_1 < c_1, c_2, c_3 \\
 b_2 < c_2, c_3 & b_2 < c_2, c_3 \\
 b_3 < c_3, c_1 & b_3 < c_1 \\
 c_1 < d_1, d_2 & c_1 < d_1, d_2, d_3 \\
 c_2 < d_2, d_3 & c_2 < d_2, d_3 \\
 c_3 < d_3, d_1 & c_3 < d_1, d_3 \\
 d_1 < e_1, e_2 & d_1 < e_1, e_2, e_3 \\
 d_2 < e_2, e_3 & d_2 < e_2, e_3 \\
 d_3 < e_3, e_1 & d_3 < e_1 \\
 e_1, e_2, e_3 < 1 & e_1, e_2, e_3 < 1
 \end{array}$$

Let $ex\Psi_P(y; a, b)$ and $ex\Psi_Q(y; a, b)$ be their respective extended ab-indices, and $ex\Psi_P(1; a, b)$ and $ex\Psi_Q(1; a, b)$ the corresponding pull-back ab-indices.

Determine the smallest coefficient of an ab-monomial in the difference $ex\Psi_P(1; a, b) - ex\Psi_Q(1; a, b)$.

Question 066
solved in Stage 2

Consider the cycle graph C on 5 vertices. Let L be the lattice of flats of the associated matroid and \mathcal{G} the subset of L given by edge sets of connected induced subgraphs. In the paper <https://arxiv.org/abs/2602.21194>, refinements of the doubled nested set fan $\Sigma^{2\mathcal{G}}$ associated to the pair (L, \mathcal{G}) are defined. The authors show that the tubing refinement of $\Sigma^{2\mathcal{G}}$ defines a subdivision of the nested set fan $\Sigma_{\mathcal{G}}$. Compute the number of maximal cones in this subdivision, using the results from <https://arxiv.org/abs/2602.21194>. How many of them are simplicial?

Question 067
solved in Stage 2

Consider the definition of the MMC regions and strata in Section 5 of the paper <https://arxiv.org/abs/2503.09571>. For $n = 15, r = 6$ and the sign pattern $\sigma = (+, +, -, +, +, +, +, +, +, -, +, +, +, +)$ how many MMC strata $C_{n,\sigma,r}^0$ of dimension 24 are there?

Question 068
solved in Stage 2

Consider variables u_{ij} labeled by chords in an n -gon with vertices $1, \dots, n$ and the relations $R_{ij} := u_{ij} + \prod_{kl \approx ij} u_{kl} - 1$ where $kl \approx ij$ means the chords do not cross. For $n = 8$ how many sign patterns in $\{+, -\}^{n(n-3)/2}$ (which assigns a sign to each variable u_{ij}) do not contradict R_{ij} ?

Question 069
solved in Stage 2

Let $ex\Psi_{\Pi_n}(y, a, b)$ be the extended ab -index of the set partition lattice Π_n of rank n . Compute $ex\Psi_{\Pi_n}(1, 1, 1)$ for $n = 12$.

Question 070
solved in Stage 2

Take a square piece of paper of thickness d and side lengths $l = 1$ with barycenter at the origin. First, fold the paper randomly and endow the two resulting sides with different topologies. Fold it back to the original structure. Now fold it three times along the longest edge, i.e., half the longest edge. All folding is performed in Euclidean space \mathbb{R}^3 .

The structure you have now will have layers on top of each other endowed with different topologies. Now take one of the vertices and fold it to the middle of the longest edge.

Now subdivide the resulting structure into a triangle and a rectangular shape. Puncture the triangle at its barycenter with a pointy steel rod. Unfold the structure again to its original shape with side length 1.

Group the holes according to their distance to the origin. Remember that you have two topologies. This gives the tuple of the cardinality of the distance classes.

Now take the length of this tuple and define it as c . Organize all holes on the paper, which has thickness d , in a cyclic graph. Let c be the number of possible colors for the edges. Compute the number of monochromatic perfect matchings of the graph.

Question 071
solved in Stage 2

Let x be a point in the complex Grassmannian $G(k, n)$, and let M be the corresponding matroid. The Chow class of M is defined as the class of the torus orbit closure $(\mathbb{C}^*)^n x$ in the Chow ring of $G(k, n)$.

Expand the Chow class of the non-nesting rook matroid (arXiv: 2410.00127) defined by $UUUURURRRRUR$ and $RUURUUURRRRUU$ as a linear combination of Schubert cycles s_λ .

Question 072
solved in Stage 2

Let f_1, \dots, f_{12} be 12 linear forms on a 6-dimensional complex vector space V which define a hyperplane arrangement that realizes the matroid FP_{12} . Let W be the subspace of rational functions on PV spanned by elements of the forms $\prod_{i \in B} f_i^{-1}$, where B is a basis of FP_{12} . Choose a generic hyperplane in V , and let X be the kernel of the restriction map from W to this generic hyperplane. Compute the dimension of X .

Question 073
solved in Stage 2

How many combinatorial types of minuscule type B_3 strong valuated Coxeter matroids are there? By 'combinatorial type', we mean the number of induced regular subdivisions of the 3-cube. Give me only the number of such matroids up to symmetry of the cube.

Question 074
solved in Stage 2

We use the notation of <https://arxiv.org/pdf/2507.02426v2>.

Let C/K be the curve defined by

$$y^2 = x(x+1)(x+5)(4x+1)(x+8)(x+8 \cdot 2^{1/2})(x+8 \cdot 2^{3/4})(x+16)(x+32)$$

What are the normalized thicknesses of the double points of the stable marked model of C in the sense of the above article? Return your final answer as a descending list in the form (a, b, c, \dots) .

Question 075
solved in Stage 2

Let X be the Segre-Veronese embedding of $Y = (\mathbb{P}^1)^6$ with respect to the line bundle $\mathcal{O}_Y(1, 2, 3, 4, 5, 6)$. Compute the Rayleigh-Ritz degree $RRdeg_{10}(X)$ with respect to a generic quadratic form.

Question 076
solved in Stage 2

Let L denote the log-canonical bundle on $\overline{M}_{0,20}$ over a field of characteristic 2. Compute the dimension of $H^{17}(\overline{M}_{0,20}, L^{-1})$.

Question 077
solved in Stage 2

Consider a triangulated surface with 11 vertices and whose facets are given by the following set of triangles:

$$\begin{aligned} &\{(1, 5, 6), (1, 2, 6), (1, 5, 7), (5, 10, 7), (9, 10, 7), (4, 7, 9), (1, 7, 3), \\ &\quad (1, 3, 4), (1, 4, 8), (1, 8, 9), (1, 9, 10), (1, 10, 2), (10, 8, 2), (10, 3, 8), \\ &\quad (10, 11, 3), (2, 3, 11), (2, 8, 4), (4, 5, 2), (6, 2, 7), (2, 11, 7), (8, 11, 7), \\ &\quad (3, 7, 8), (6, 7, 4), (6, 4, 3), (6, 3, 9), (6, 9, 8), (6, 8, 11), (6, 11, 5), \\ &\quad (11, 9, 5), (2, 3, 5), (9, 4, 11), (10, 11, 4), (4, 5, 10), (3, 9, 5)\}. \end{aligned}$$

To begin, find coordinates for each of the 11 vertices, such that the topology of the surface is correct (its genus is the right one and facets overlap only on edges or vertices or the empty set; it has to be an embedded simplicial complex) and every triangle is flat (inside a hyperplane and is the convex hull of its vertices). This initial auxiliary answer should be a 3×11 matrix A of integer numbers. Finally, multiply the A you obtained by $y = [1, 1, 1]$, $yA = B$ is a 1×11 list of integer numbers. Call D the number of distinct numbers you get in B . Multiply D by the genus of the surface. Print that final number.

Question 078
solved in Stage 2

What is the number of equivalence classes of d -dimensional virtual polytopes with $d + 1$ vertices up to affine equivalence for $d = 2$?

Question 079
solved in Stage 2

Find the number of vertices of the reflexive flow polytope of the complete directed graph with six vertices.

Question 080
solved in Stage 2

Let K be the knot 9_{46} , n an integer and M_n the Dehn filling of K along the slope $1/n$. The two strong inversions of K extend to M_n . Let K_n^1 and K_n^2 be the branch sets in the respective quotients. For $i = 1, 2$, let g_i be the limit of $Kh(K_n^i) - Kh(K_{n-1}^i)$ as n goes to ∞ , where $Kh(L)$ denotes the (unreduced) Khovanov homology of a link L . What is the dimension of the (hat version of) Heegaard Floer homology of $M_{g_1 \cdot g_2}$? All homology theories are taken with coefficients in the field of 2 elements.

Question 081
solved in Stage 2

Consider the moduli space of 7 labeled points in linearly general position in the projective plane. This is the open Grassmannian $\text{Gr}(3, 7)$ where all 35 Plücker coordinates are non-zero modulo the action of the 7-dimensional torus. We are interested in the log canonical embedding of this moduli space. What is the dimension of the projective space where this log canonical model lives?

Question 082
solved in Stage 2

Let G be an undirected graph with d vertices. Let \mathcal{I}_G be the set of global Markov statements $A \perp B | S$ for G (where “ \perp ” stands for “independent”) such that the collection $\{A, B, S\}$ is a partition of V . For any vector $x \in \mathbb{R}^d$ and any subset $S \subseteq [d]$, we denote by x_S the projection of x onto the coordinates with indices in S . Fix a subset $\mathcal{X} \subseteq \mathbb{R}^d$ and define $\mathcal{P}_G^{(0)} := \text{conv}(\mathcal{X}) \subseteq \mathbb{R}^d$, where “conv” stands for convex hull. For $i \geq 1$ define

$$\mathcal{P}_G^{(i)} := \text{conv} \bigcup_{A \perp B | S \in \mathcal{I}_G} \{(x_A, y_B, x_S), (y_A, x_B, x_S) \mid (x_A, x_B, x_S), (y_A, y_B, x_S) \in \mathcal{P}_G^{(i-1)}\}.$$

Suppose now that $d = 4$ and that \mathcal{X} is the set of 10 points in \mathbb{R}^4 whose coordinates are the columns of the matrix

$$\begin{pmatrix} \frac{10}{9} & 1 & 1 & \frac{4}{5} & \frac{4}{5} & \frac{3}{2} & \frac{5}{9} & \frac{1}{5} & \frac{8}{3} & \frac{2}{5} \\ \frac{3}{5} & \frac{6}{5} & 4 & \frac{5}{2} & \frac{1}{9} & 6 & \frac{3}{4} & 2 & 1 & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{3}{5} & 3 & \frac{4}{3} & \frac{2}{3} & \frac{1}{3} & \frac{4}{5} & 2 & \frac{1}{2} \\ 2 & 1 & \frac{2}{5} & 1 & \frac{8}{9} & 1 & 1 & 1 & \frac{9}{4} & 2 \end{pmatrix}.$$

Determine the number of vertices of $\mathcal{P}_G^{(2)}$ with respect to the 4-cycle G .

Question 083
solved in Stage 2

Consider the 5-dimensional cube C_5 and all the non-simple 4-dimensional combinatorial types of slices of C_5 obtained by intersecting C_5 with a hyperplane H . How many of these are not combinatorially equivalent to a product of the form $(C_4 \cap H) \times C_1$?

Question 084
solved in Stage 2

Fix $n \in \mathbb{N}$, $r \in [n]$, $q \geq 4$, and $0 < t_1 < \dots < t_n$. Set $X_n := 2^{\{t_1, \dots, t_n\}}$. For $c = (c_1, \dots, c_p) \in \mathbb{R}^p$, define $\Psi_c : X_n^p \rightarrow X_n$ as follows. Given $E = (E_1, \dots, E_p)$, write

$$\bigcup_{j=1}^p E_j = \{s_1 < \dots < s_a\}.$$

Let $z_0 := 0$, and for $i = 1, \dots, a$ put

$$z_i := \mathbf{1}_{\{z_{i-1} \leq 1\}} \cdot (z_{i-1})_+ + \sum_{j=1}^p c_j \mathbf{1}_{\{s_i \in E_j\}}.$$

Then

$$\Psi_c(E) := \{s_i : 1 \leq i \leq a, z_i > 1\}.$$

Let

$$\alpha = (K, n_1, \dots, n_{K-1}, B^{(1)}, \dots, B^{(K)}), \quad n_0 = n_K = 1,$$

with $B^{(\ell)} \in \mathbb{R}^{n_\ell \times n_{\ell-1}}$. If $b_i^{(\ell)}$ denotes the i -th row of $B^{(\ell)}$, define

$$G_\alpha^{(0)}(E) := E, \quad G_\alpha^{(\ell)}(E) := (\Psi_{b_1^{(\ell)}}(G_\alpha^{(\ell-1)}(E)), \dots, \Psi_{b_{n_\ell}^{(\ell)}}(G_\alpha^{(\ell-1)}(E))).$$

Impose $\max_{\ell, j} \#\{i : B_{ij}^{(\ell)} \neq 0\} \leq q$. For $\rho \in [n]$, let $\mathcal{G}_{n, \rho, q}$ be the class of all maps $g : X_n \rightarrow X_n$ for which there exists an α satisfying the above constraint such that $g = G_\alpha^{(K)}$ and $|g(\{t_1, \dots, t_n\})| \leq \rho$. Define

$$\mathfrak{N}(n, \rho, q) := \sup_{g \in \mathcal{G}_{n, \rho, q}} \min_{\substack{\alpha : G_\alpha^{(K)} = g \\ \max_{\ell, j} \#\{i : B_{ij}^{(\ell)} \neq 0\} \leq q}} \sum_{\ell=1}^K n_\ell$$

and

$$\mathfrak{S}(n, \rho, q) := \sup_{g \in \mathcal{G}_{n, \rho, q}} \min_{\substack{\alpha : G_\alpha^{(K)} = g \\ \max_{\ell, j} \#\{i : B_{ij}^{(\ell)} \neq 0\} \leq q}} \#\{(\ell, i, j) : B_{ij}^{(\ell)} \neq 0\}.$$

Determine the asymptotic order, up to polylogarithmic factors, of

$$\mathfrak{N}(n, n, q) + \mathfrak{S}(n, r, q).$$

Question 085
solved in Stage 3

Consider the 6-dimensional linear space of symmetric matrices $L = \text{Span}\{A_1, \dots, A_6\} \subset \mathbb{C}^{6 \times 6}$ generated by

$$\begin{aligned}
 A_1 &= \begin{bmatrix} 25 & 48 & -14 & 18 & 10 & 13 \\ 48 & -2 & -46 & 36 & -18 & -6 \\ -14 & -46 & -2 & -6 & -5 & -16 \\ 18 & 36 & -6 & 28 & 7 & 12 \\ 10 & -18 & -5 & 7 & -22 & -1 \\ 13 & -6 & -16 & 12 & -1 & -11 \end{bmatrix}, & A_2 &= \begin{bmatrix} 48 & -2 & -46 & 36 & -18 & -6 \\ -2 & 89 & 9 & 23 & 47 & 43 \\ -46 & 9 & 33 & -20 & 34 & 12 \\ 36 & 23 & -20 & 33 & -11 & 8 \\ -18 & 47 & 34 & -11 & 19 & 20 \\ -6 & 43 & 12 & 8 & 20 & 27 \end{bmatrix}, \\
 A_3 &= \begin{bmatrix} -14 & -46 & -2 & -6 & -5 & -16 \\ -46 & 9 & 33 & -20 & 34 & 12 \\ -2 & 33 & -1 & 12 & 25 & 14 \\ -6 & -20 & 12 & -26 & -21 & -6 \\ -5 & 34 & 25 & -21 & -1 & 10 \\ -16 & 12 & 14 & -6 & 10 & 18 \end{bmatrix}, & A_4 &= \begin{bmatrix} 18 & 36 & -6 & 28 & 7 & 12 \\ 36 & 23 & -20 & 33 & -11 & 8 \\ -6 & -20 & 12 & -26 & -21 & -6 \\ 28 & 33 & -26 & 27 & 10 & 10 \\ 7 & -11 & -21 & 10 & 6 & -2 \\ 12 & 8 & -6 & 10 & -2 & 0 \end{bmatrix}, \\
 A_5 &= \begin{bmatrix} 10 & -18 & -5 & 7 & -22 & -1 \\ -18 & 47 & 34 & -11 & 19 & 20 \\ -5 & 34 & 25 & -21 & -1 & 10 \\ 7 & -11 & -21 & 10 & 6 & -2 \\ -22 & 19 & -1 & 6 & 47 & 11 \\ -1 & 20 & 10 & -2 & 11 & -7 \end{bmatrix}, & A_6 &= \begin{bmatrix} 13 & -6 & -16 & 12 & -1 & -11 \\ -6 & 43 & 12 & 8 & 20 & 27 \\ -16 & 12 & 14 & -6 & 10 & 18 \\ 12 & 8 & -6 & 10 & -2 & 0 \\ -1 & 20 & 10 & -2 & 11 & -7 \\ -11 & 27 & 18 & 0 & -7 & 62 \end{bmatrix}.
 \end{aligned}$$

Let $X \subseteq L$ be the subvariety of matrices of rank ≤ 3 . What is the number of irreducible components of X ?

Question 086
solved in Stage 3

Let KQ be the path algebra where Q is of linearly oriented Dynkin type A_3 having 3 vertices. Let A be the total preprojective algebra of KQ . How many indecomposable non-zero A -modules are there up to isomorphism?

Question 087
solved in Stage 3

Let KQ be the path algebra of the Dynkin quiver of type A_5 (so Q has 5 vertices) with linear orientation. Let A be the Koszul dual algebra of the Auslander algebra of KQ . How many indecomposable spherical A -modules are there? Here a module M is called spherical (in the sense of Auslander-Bridger) if M has finite projective dimension equal to n and $\text{Ext}_A^i(M, A) = 0$ for $i = 1, \dots, n-1$ with the convention that every module of projective dimension less than or equal to 1 is spherical.

Question 088
solved in Stage 3

Let A be the connected linear Nakayama algebra with Kupisch series $[2, 2, 2, 2, 1]$ given by quiver and relations over a field. How many spherical indecomposable A -bimodules are there? Here a module M over an algebra B is called spherical if M has finite projective dimension n and $\text{Ext}_B^i(M, B) = 0$ for $i = 1, \dots, n-1$. By convention all modules of projective dimension ≤ 1 is spherical. For this problem, being spherical is meant in the bimodule sense.

Question 089
solved in Stage 3

Let Q be the quiver on the vertex set $\{v_1, \dots, v_6\}$ with arrows

$$\begin{aligned}
 a_1: v_1 &\rightarrow v_2, & a_2: v_2 &\rightarrow v_3, & a_3: v_2 &\rightarrow v_5, \\
 a_4: v_3 &\rightarrow v_1, & a_5: v_3 &\rightarrow v_4, & a_6: v_4 &\rightarrow v_2, \\
 a_7: v_4 &\rightarrow v_6, & a_8: v_5 &\rightarrow v_4, & a_9: v_6 &\rightarrow v_3.
 \end{aligned}$$

Let $A = \mathbb{F}_3 Q / I$, where \mathbb{F}_3 is the field with three elements and I is the two-sided ideal of the path algebra $\mathbb{F}_3 Q$ generated by

$$\begin{aligned}
 a_2 a_4, & \quad a_2 a_5 + 2 a_3 a_8, & a_4 a_1 + a_5 a_6, & \quad a_6 a_3, & \quad a_6 a_2 + 2 a_7 a_9, \\
 a_8 a_6, & \quad a_9 a_4, & \quad a_9 a_5, & \quad a_4 a_1 a_2, & \quad a_1 a_2 a_5 a_7.
 \end{aligned}$$

How many indecomposable A -modules are there?

Question 090*solved in Stage 3*

Let A be a Nakayama algebra with n simples, that is an algebra given by quiver and relations over a field where the quiver is a directed cycle of length n and the relations are monomial relations possibly of length 1, i.e. connected cyclic Nakayama algebras and (possibly not connected) linear Nakayama algebras. Determine the number of such Nakayama algebras with 20 simples that have even global dimension and are higher Auslander algebras, i.e. where the global dimension and the dominant dimension of the algebra coincide.

Question 091*solved in Stage 3*

We color each element of the set $\{1, 2, \dots, 1000\}$ red or blue. Then we arrange the red elements in a Standard Young Tableau S and the blue elements in a Standard Young Tableau T so that the column lengths of both S and T come from the set $\{2, 4, 6, 8, 10\}$. In how many ways is this possible? Provide the first ten digits and the last ten digits of the answer.

Question 092*solved in Stage 3*

Let $[L]$ be the point of the Grassmannian $Gr(14, 105)$ corresponding to the 14-dimensional braid arrangement, and let X be the torus-orbit closure of $[L]$. Compute the coefficient of the Schubert variety corresponding to the partition $(91)^{12}, 78, 0$ in the expansion of the homology class of X in the Schubert basis.

Question 093*solved in Stage 3*

Johnny's a farmer. Johnny has antlers, birds and cats on his farm: At least one cat, more birds than cats and more antlers than birds, and he loves all of them dearly. Johnny likes counting. One day he counts his cats, cubes the number and writes it down in his diary. The next day he counts the birds and the cats. He computes both the sum of cats and birds and the product of cats and birds and multiplies them with each other. The result he writes in his diary. On the third day, he counts the birds, cubes the number and writes it in his diary. On the fourth day, he counts the antlers and the cats. He calculates both the sum of cats and antlers and the product of cats and antlers and multiplies them with each other. The result he writes in his diary. On the fifth day, he counts antlers, birds and cats, multiplies their numbers and writes down the result times 3. On the sixth day, he counts antlers and birds. He calculates both the sum of birds and antlers and the product of birds and antlers and multiplies them with each other. The result he writes in his diary. On the seventh day, Johnny only counts the antlers and writes down the cube of their number into his diary. He then sees that it is good, because all seven numbers in his diary add up to 4 times the sum of birds and cats, times the sum of antlers and birds, times the sum of antlers and cats. How many antlers, birds and cats are on Johnny's farm? Find the solution with the smallest total number of animals. Answer with the 3-tuple (#antlers, #birds, #cats), where #antlers denotes the number of antlers (likewise for the other animals).

Question 094*solved in Stage 3*

What is the maximum number of vertices of a hyperplane section of an octahedron? A hyperplane section is the intersection with a hyperplane.

Question 095*solved in Stage 3*

Let K_1 and K_2 be any alternating knots with Seifert genus 1 and signature 0. Suppose that $\det(K_1) = 2001$ and $\det(K_2) = 2025$. Let M_i be the exterior of K_i for $i = 1, 2$. Let M be the manifold obtained by gluing M_1 to M_2 along their boundaries, identifying the meridian of one knot with the Seifert longitude of the other knot and vice versa. What is the dimension of the hat version of Heegaard Floer homology of M with coefficients in the field of two elements?

Question 096

solved in Stage 3

For n a positive integer define $V(n)$ to be the integer obtained by using the base 10 digits of n in base 11. I want to evaluate the series $\sum_{p \text{ prime}} \frac{1}{V(p)}$ to an accuracy of 10^{-5} .

Question 097

solved in Stage 3

Let

$$I = \frac{1}{\pi^2} \int_0^\pi \int_0^\pi \log |1 + 2e^{ix} + 3e^{iy}| \, dy \, dx.$$

Evaluate I in closed form as a finite expression built from explicit complex numbers, Riemann zeta values, one-variable classical polylogarithm values ($\text{Li}_s(q)$, where $s \in \mathbb{Z}_{>0}$ and $q \in \mathbb{C}$), and the operations $+$, $-$, \times , \div .

Question 098

solved in Stage 3

In terms of d the dimension, what is the ratio of the surface area of a d -dimensional ℓ_p ball to its volume?

Question 099

remains unsolved

Let Q be the quiver with 4 vertices, with adjacency matrix $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 0 \end{pmatrix}$, and let $d = (1, 3, 4, 1)$.

Count all the unique Luna types for representations of Q of dimension vector d , across all stability conditions that admit semistables.

Question 100

remains unsolved

Let P be a set of 6 permutations of size n such that the identity and the reverse permutation are elements of P . What is the maximum density of triples shattered by P asymptotically as $n \rightarrow \infty$?

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